International Journal of Theoretical Physics, Vol. 35, No. 1, 1996

Matter Production in the Early Universe

M. Mattes,[1] U. Ochs,[1] and M. Sorg[1]

Received March 12, 1995

The coupled Dirac–Einstein equations with a negative cosmological constant for an open FRW universe are studied in detail. The corresponding solutions admit bounces ($\rightsquigarrow$ minimal radius) of the universe such that the matter energy in any comoving 3-volume is either increased or decreased during the bounce according to whether the bounce pressure of the spinor field is appropriately negative or not. If matter is generated (annihilated) during a bounce, the universe subsequently becomes larger (smaller) than before the bounce. Therefore matter can be generated only during the growth of the universe, but it is annihilated again during the subsequent shrinking phase, which together with the growing phase forms a cosmic supercycle.

1. INTRODUCTION

Concerning the cosmic evolution of our universe, much attention is currently being given to inflationary scenarios because many (if not all) of the notorious difficulties with the standard cosmological model can be overcome within the framework of that new paradigm [for reviews see Blau and Guth (1987) and Linde (1987); also see Abbott and Pi (1986), Dolgov *et al.* (1990), and Kolb and Turner (1990)]. However, there is also some doubt whether inflation really is a satisfactory solution of the old cosmological puzzles, the objections being partly of technical (Hawking, 1990; Penrose, 1990) and partly of a more philosophical nature (Penrose, 1989) or they are simply a matter of taste (Dicke, 1990). As to the technical side, it is no problem to conceive several alternative foundations of the inflation mechanism (Mattes and Sorg, 1991, 1992), provided one is convinced that inflation per se is the true explanation for the past evolution of the universe. However, if one does not like the very idea of inflation, one will perhaps prefer the model of an oscillating universe (Dicke, 1990), where the universe starts with a small

[1] II. Institut für Theoretische Physik der Universität Stuttgart, D-70550 Stuttgart, Germany.

size and with few particles (perhaps a single one) but becomes larger and equipped with ever more energy and particles through any one of the consecutive cosmic cycles. Now it may seem as if the latter point of view would require a rather exotic physics for generating the desired oscillations, whereas inflation is based upon a seemingly plausible idea, namely the emergence of some weakly interacting scalar field within the framework of the Grand Unified Theories (Turner, 1986).

However, we want to demonstrate in the present paper that just the opposite is true: for an oscillating universe model (of the kind mentioned above) there are needed only two ingredients: (i) Einstein's equations with a negative cosmological constant and (ii) Dirac's equation. Evidently, these two building blocks form a less speculative basis than is required for the inflationary paradigm, because the existence of the Dirac spinor field in nature is beyond any doubt, in contrast to that scalar field of questionable origin (Turner, 1986).

In contrast to the scalar fields, the physical relevance of Dirac's spinor field has been well established in many areas of physics (e.g., atomic spectra, scattering phenomena, etc.), and this success may be referred to both the *classical* and the *quantized* versions of the spinor field. Moreover, it is generally presumed that the most elementary constituents of matter (quarks and leptons) are also Dirac particles. In view of this overwhelming significance of spinor fields, it seems reasonable to assume that these fields will play also a dominant role for the description of the matter in the very early universe when it was not yet filled with a *hot gas of decoherent particles.* (Such a thermodynamic state of matter would produce a *positive* value for pressure $\mathcal{P}$ and energy density $\mathcal{M}$ and therefore would lead to the inconsistencies of the standard cosmological model mentioned above.) During the prethermodynamic phase, the universe is imagined to be filled by a globally coherent quantum state of a Dirac spinor field which can produce also a *negative pressure* acting as the origin of *both* the universe's outward push *and* the generation of matter-energy. Thus we are led to the idea that matter-energy was created by the negative pressure during the prethermodynamic phase in the form of a globally coherent spinor field obeying Dirac's *classical* field equation. Such a classical field configuration (extending globally over the tiny but rapidly expanding universe) cannot be identified with some ordinary quantum state which is occupied by a definite particle number such as, e.g., the electron's ground state in the hydrogen atom ($\rightsquigarrow$ Pauli's exclusion principle). The reason is that those ordinary quantum states subject to the exclusion principle must be identifiable uniquely by fixed quantum numbers (charge, energy, angular momentum, etc.). However, the violent expansion dynamics of the primeval space-time does not admit the spatial normalization of certain conserved quantities carried by that spinor field and therefore the notion of particle number cannot be applied in its conventional meaning. Alternatively, we will introduce the concept of "particle number μ" by simply measuring the matter-energy of the classical spinor field in units of the rest energy Mc^2 of a conventional Dirac particle. This number μ is then considered as a rough estimate of the number of decoherent ordinary fermions which are released through the transition of the global quantum state into the hot thermodynamic phase (as the starting point of the standard cosmology). However, the precise nature of that phase transition is a controversial point in the literature and we want to restrict ourselves to the question of whether the coupled Dirac–Einstein dynamics of the expanding universe is capable of producing large enough values for the "particle number μ" during the *prethermodynamic* phase (the number of present-day fermions is usually thought to be of magnitude $\gtrsim 10^{80}$).

In some previous papers, the cosmological implications of the coupled Dirac–Einstein equations (without a cosmological constant) have already been studied (Sorg, 1992a,b, 1994; Ochs and Sorg, 1993, 1994; Mattes and Sorg, 1993) and it has been found that the *closed* (FRW) universe must be excluded. The *flat* case yields the ordinary, matter-dominated universe of the old standard model, which is plagued with its well-known deficiencies. Therefore, it is exclusively the *open* case which is nontrivial and exhibits some interesting properties (Ochs and Sorg, 1993): (i) there is a soft birth of the universe (vanishing matter content $\Rightarrow$ "creation *ex nihilo*"), (ii) matter is generated during a short inflation-like phase, where the "radius" of the universe is not yet appreciably greater than the Compton wavelength of the Dirac particle, and (iii) there are also bounce solutions, where the radius of the universe drops below the Compton wavelength ($\Rightarrow$ quantum effects) and the universe's energy content changes suddenly.

However, despite these welcome features of the original Dirac–Einstein equations, there is also one deficiency which motivates us to look for a further improvement: this is the fact that the production of matter during the short inflation-like phase is too weak to account for the huge matter content of the universe observed today. For instance, if the young universe initially contains one single particle per comoving 3-cell (of Compton length), then the final number of particles in this cell never is greater than three (Section 2). Because of this unrealistic prediction one has to look for a matter-producing modification of the Dirac–Einstein model, which, however, can be achieved in a very simple way (Section 3): one merely has to include a cosmological term (with a *negative* cosmological constant). The reason is that for bounded particle number per comoving 3-cell such a term forbids the infinite growth of the universe's radius $\mathfrak{R}$ even in the open case, so that the extension of the universe becomes bounded from above. On the other hand, when the universe has passed its maximal extension and the radius $\mathfrak{R}$ has become very small again, the negative pressure of the Dirac spinor field blows up the universe and thus prevents it from collapsing (Section 4). The result of these two effects is the emergence of oscillations of a special kind, where the matter content is changing suddenly during that short time period when the radius is of the order of magnitude of the Compton wavelength. Fortunately, the matter content *increases* when the maximal extension of the universe is *growing* during the consecutive cosmic cycles (and vice versa).

We elaborate and discuss these effects by means of some detailed numerical computations.

2. EQUATIONS OF MOTION

As mentioned above, the present model is based upon the minimal coupling of Dirac's equation

$$i\hbar c\gamma^{\mu}\mathcal{D}_{\mu}\psi = Mc\psi \tag{1}$$

to gravity; i.e., we complement this field equation for the wave function ψ by Einstein's equation with a cosmological term:

$$E_{\mu\nu} = 8\pi \frac{L_p^2}{\hbar c} (T_{\mu\nu} + \lambda_0 G_{\mu\nu}) \tag{2}$$

$$\left(\text{Einstein tensor} \quad E_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} RG_{\mu\nu}\right)$$

This cosmological complement has been included in such a way that it may be considered as an extra energy-momentum density which is normally attributed to the physical vacuum (Weinberg, 1989).

The matter energy-momentum density $T_{\mu\nu}$ is assumed to be exclusively due to the spinor field ψ and must exhibit the cosmological shape

$$T_{\mu\nu}[\psi] = \mathcal{M}[\psi]b_{\mu}b_{\nu} - \mathcal{P}[\psi]\mathcal{B}_{\mu\nu} \tag{3}$$

where the unit vector b_{μ} points in the direction of cosmic time θ

$$b_{\mu} = \partial_{\mu}\theta \tag{4}$$

$$(b_{\mu}b^{\mu} = +1)$$

and the projector $\mathcal{B}_{\mu\nu}$ acts orthogonal to this time direction

$$b^{\mu}\mathcal{B}_{\mu\nu} = 0 \tag{5a}$$

$$\mathcal{B}_{\mu\nu}\mathcal{B}^{\nu}_{\lambda} = \mathcal{B}_{\mu\lambda} \tag{5b}$$

Clearly, it is not at all obvious that the energy-momentum density $T_{\mu\nu}$ of the Dirac field ψ obeys the cosmological shape (3), because it carries a nonvanishing spin density which, in general, will break the exact Robertson–Walker symmetry. However, the Dirac equation on a Robertson–Walker background admits a certain subclass of solutions producing an energy-momentum density $T_{\mu\nu}$ of the exact cosmological form (3) (Sorg, 1992a, Ochs and Sorg, 1994). We restrict ourselves to this subclass of solutions throughout this paper, where $T_{\mu\nu}$ does not explicitly contain the spin effects.

If the cosmological term is also put into the physical form (3), one finds a somewhat exotic equation of state for the vacuum, namely

$$\mathcal{P}_0 = -\mathcal{M}_0 = -\lambda_0 \; (= \text{const}) \tag{6}$$

Similarly, the special configurations of Dirac's spinor field ψ mentioned above produce energy density $\mathcal{M}$ and pressure $\mathcal{P}$ of the following kind (Sorg, 1992a):

$$\mathcal{M} = 3\hbar c\rho\left(\frac{\sigma}{2\mathcal{R}}\cos(\chi) + \frac{m}{3}\right) \tag{7a}$$

$$\mathcal{P} = \hbar c\rho\,\frac{\sigma}{2\mathcal{R}}\cos(\chi) \tag{7b}$$

$$(\text{open universe: } \sigma = +1)$$

Here, $\mathcal{R}$ is the "radius" of the FRW universe, for which we consider only the open case ($\sigma = +1$). Further, $m = Mc/\hbar$ denotes the inverse Compton wavelength of the Dirac particle, whose wave function ψ has been parametrized by the scalar density $\rho = \bar{\psi}\psi$ and by the relative phase angle χ of the positive- and negative-energy components (Mattes and Sorg, 1993). Fortunately, the energy-momentum density $T_{\mu\nu}$, (3), does not contain the remaining six physical parameters (besides ρ and χ) building up the eight-component wave function ψ.

Of course, Dirac's equation (1) for the wave function ψ must yield now the equations of motion for its parametrizations χ and ρ (=: $\mu/\mathcal{R}^3$), i.e.,

$$\dot{\chi} = \sigma\left(2 + 3\,\frac{\cos(\chi)}{r}\right) \tag{8a}$$

$$\dot{\mu} = 3\,\frac{\sigma}{r}\,\mu\,\sin(\chi) \tag{8b}$$

where cosmic time θ and the radius $\mathcal{R}$ have been rescaled by the Compton length (i.e., $\theta \rightsquigarrow t = m\theta$; $\mathcal{R} \rightsquigarrow r = m\mathcal{R}$). The last dynamical equation is that for the radius r and is found from the Einstein equation (2) as

$$\ddot{r} = -\frac{4\pi}{3}\Lambda^2\frac{\mu}{r^2}\left(1 + 3\sigma\frac{\cos(\chi)}{r}\right) + \frac{8\pi}{3}\Lambda^2 k_0 r \tag{9a}$$

$$\dot{r}^2 = \sigma + 8\pi\Lambda^2\frac{\mu}{r}\left(\frac{\sigma}{2r}\cos(\chi) + \frac{1}{3}\right) + \frac{8\pi}{3}\Lambda^2 k_0 r^2 \tag{9b}$$

We have preferred to work here with the dimensionless constant Λ ($=mL_p$) and with k_0 [$=\lambda_0 m^{-4}(\hbar c)^{-1}$] in place of the cosmological constant λ_0. For vanishing cosmological term ($k_0 \to 0$), the generalized system (9a), (9b) is reduced to the previous case of Ochs and Sorg (1993).

The systems of equations (8) and (9) are the coupled Dirac–Einstein equations for an FRW universe filled with Dirac's spinor field, and we are going to study the corresponding solutions in some detail. But before getting involved with the numerical computations, it is always useful to look for a qualitative picture of what is to be expected. To this end, first remember (Ochs and Sorg, 1993) that for $k_0 = 0$, i.e., for vanishing cosmological term, the radius r is growing infinitely, i.e., asymptotically as $r(t) \cong t$ for $t \to \infty$, as can be read off directly from the so-called "initial-value equation" (9b). However, it is just this equation which forbids such an asymptotic behavior for negative cosmological constant ($k_0 < 0$), provided there is negligible production of matter ($\Rightarrow \mu \approx$ const). Assuming that the maximal radius r_{max} ($\dot{r}|_{max} = 0$) is very large, we conclude from that equation for *negligible increase* of particle number μ (according to the standard model)

$$r_{max} \approx \left(\frac{8\pi}{3}\Lambda^2|k_0|\right)^{-1/2} \tag{10a}$$

but obtain in the case of *violent particle production* up to μ_{max}

$$r_{max} \approx \left(\frac{\mu_{max}}{|k_0|}\right)^{1/3} \tag{10b}$$

In the first case (10a), the radius r is strictly bounded from above and the universe collapses to a point again (Rindler, 1977) (big crunch). In the second case (10b), being admitted exclusively by the present model, the big crunch might well be avoided (see below) because the "creation-*ex-nihilo*" solutions (Ochs and Sorg, 1993) persist also for nonvanishing k_0 in the form ($t << 1$)

$$r(t) = t + \frac{2\pi}{18}\Lambda^2(\mu_c + 4k_0)t^3 + \cdots \tag{11a}$$

$$\chi(t) = \frac{\pi}{2} + \frac{1}{2}t + \cdots \tag{11b}$$

$$\mu(t) = \mu_c t^3 + \cdots \tag{11c}$$

and consequently the inverse process, "extinction into *nihilo*," must also be possible, in place of the singular crunch. Furthermore, the system has some tendency to avoid even such a soft extinction, which may be seen by the following argument: whenever the radius becomes extremal (r_{ex}), the equations (9a), (9b) predict for that extremal situation

$$\ddot{r}_{ex} = \frac{\sigma}{r_{ex}} + \frac{16\pi}{3}\Lambda^2 k_0 r_{ex} + \frac{4\pi}{3}\Lambda^2\frac{\mu_{ex}}{r_{ex}^2} \tag{12}$$

i.e., when the extremal radius is very small ($r_{ex} << 1$), it must be a minimum ($\ddot{r}_{ex} > 0$), whereas for large extremal value ($r_{ex} >> 1$) one will encounter a maximum ($\ddot{r}_{ex} < 0$). Thus, we expect an oscillatory behavior of the universe, either lasting forever or up to the point of "extinction into *nihilo*."

For the special situation of both "creation *ex nihilo*" and "extinction into *nihilo*" there must occur a finite number of cosmic cycles, whose precise computation is a problem in itself (not dealt with in the present paper). Subsequently, we are mainly interested in the question of the extent to which the "particle number" μ in a comoving 3-cell (of dimension r) can be increased through continuous bouncing.

3. LIMITED PARTICLE NUMBER

In standard cosmology the number of (noninteracting) particles in any comoving 3-cell remains constant; consequently the particle density must have been infinite at the moment of creation when the radius r was (very close to) zero. Of course, predictions of this kind are not very trustworthy and there are numerous attempts to overcome this notorious singularity problem of the standard cosmological model.

The open Dirac–Einstein model, established recently (Sorg, 1992a), yields a much more reasonable result in this respect, namely by predicting the following law for the production of particles:

$$\mu(t) = \mu_* \exp\left[-3\int_t^\infty \frac{\sin\chi(t')}{r(t')}\,dt'\right] \tag{13}$$

Thus, in view of the asymptotic expansions for the radius r, (11a), and angle χ, (11b), one recovers the *nonsingular* asymptotic law (11c) for the particle number μ, with the "Compton number" μ_C emerging as a functional of the whole history ($0 \leq t \leq \infty$). But for the actual numerical integration of the system (8a)–(9b), the Compton number μ_C acts as an integration constant and now the most interesting question in the present contest is how many particles μ_* have been ultimately produced ($t \rightarrow \infty$) if the universe starts with μ_C particles within a Compton volume shortly after its birth ($t \approx 1$). In other words, we have to look for the function $\mu_* = \mu_*(\mu_C, \Lambda)$.

However, the (numerical) result *for vanishing cosmological constant* ($k_0 = 0$) is somewhat disappointing (see Fig. 1). For all values of the parameter Λ, the final particle number μ_* is bounded here by $\mu_* \lesssim 2.5\mu_C$; i.e., the final particle number μ_* is in general even less than the initial one (μ_C), at least for large enough values of the parameter Λ. Thus, the original Dirac–Einstein model (with $k_0 = 0$) fails to account for the effective production of particles, despite the admittance of the "creation-*ex-nihilo*" solutions (11).

However, there is also a second type of solution, namely the bounce solutions, where only a single bounce is occurring and the particle number is changing during that single bounce. Obviously, this offers the possibility of achieving a larger particle number by continuously repeating the single bounce with its limited individual increase of particle number. One merely has to look for a modification of the original Dirac–Einstein model which ensures the continuous repetition of the single bounce.

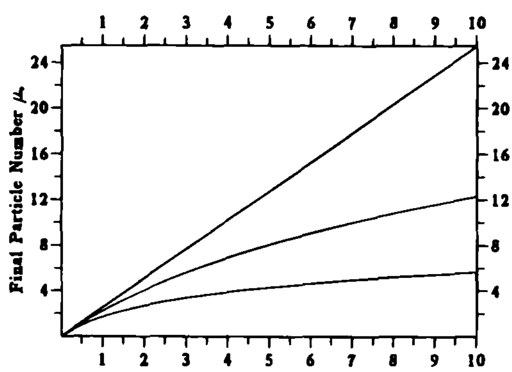


Fig. 1. Final particle number μ_*. Integrating numerically the equations of motion (8a)–(9b) for $k_0 = 0$ yields the final particle number μ_* as a function of the integration parameter μ_C. (Upper curve: $\Lambda = 0$; middle: $\Lambda = 0.25$; lower: $\Lambda = 0.5$). The upper case ($\Lambda = 0$) simultaneously is the tangent to the lower curves ($\Lambda > 0$) for $\mu_C \rightarrow 0$ and is given roughly by $\mu_* \approx 2.5\mu_C$ [in Ochs and Sorg (1993) this linear approximation has been underestimated through $\mu_*|_{\Lambda=0} \approx 0.7\mu_C$; cf. equation (55) of that paper].

4. MORE MATTER

As shown by the qualitative arguments leading to equations (10), the modification of the original equations of motion by adding the cosmological term should imply the desired effect of producing many bounces. Indeed, this expectation is well verified by the numerical solutions (Fig. 2a). Here, the maximal size r_{ex} during a single cycle does not support the first supposition (10a), applying to the standard cosmological model (where μ = const), but rather supports the second possibility (10b) on account of the violent matter production (Fig. 2b): $\mu_{max} > 10^2\mu_C$, in contrast to the situation with vanishing cosmological term ($\mu_* \lesssim 2.5\mu_C$; cf. Fig. 1).

Clearly, the origin of matter production during a bounce can be traced back to the occurrence of negative pressure $\mathcal{P}$, (7b), for the spinor field: the phase angle χ must be in the interval $\pi/2 < \chi < 3\pi/2$ during a bounce ($r \approx r_b$), but in a somewhat asymmetric way, such that the matter annihilation during the contractive phase ($\dot{r} < 0$) is overcompensated by matter production

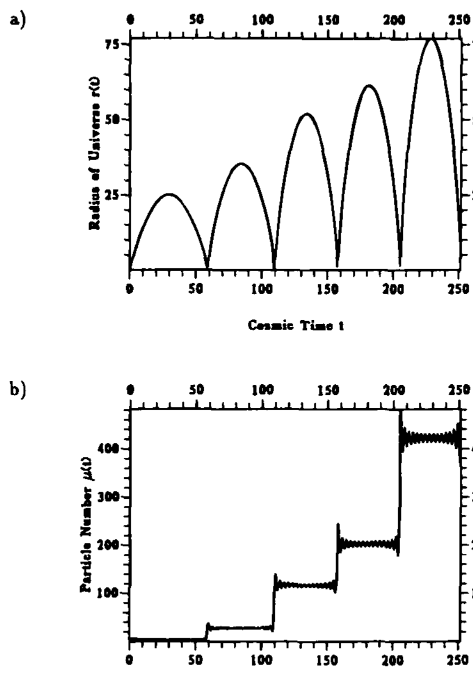


Fig. 2. Matter production by continuous bouncing. (a) A negative cosmological constant (here $k_0 = -0.001$) enforces continuous bouncing because the open universe can no longer escape to infinity for bounded particle number μ. (b) The particle number μ is increased during any bounce, but is held (approximately) constant through the remainder of the cosmic cycle (the choice for Λ was 0.5, $\mu_C = 0.1$).

in the subsequent expansive phase ($\dot{r} > 0$). Thus, we expect the occurrence of matter production during a bounce whenever the bounce angle is in the first half (mod 2π) of the interval (i.e., $\pi/2 < \chi < \pi$) and matter annihilation when it is in the second half ($\pi < \chi < 3\pi/2$); see Ochs and Sorg (1993).

When matter production is so effective, is there also annihilation of matter? The original Dirac–Einstein model admitted single-bounce solutions of two types: particle number increasing and decreasing. Therefore we have to ask: is the second type also present after inclusion of the cosmological term? Figure 3 shows that the answer is yes. The maximal radius of the seventh cycle is smaller than that of the neighboring cycles (Fig. 3a) and simultaneously the particle number during the seventh cycle is also smaller (Fig. 3b). However, the maximal radius of the fifth cycle is greater than all the preceding ones (Fig. 3a) and a similar statement holds for the particle number μ (Fig. 3b). This indicates the existence of a close correlation between the maximal radius per cycle and the corresponding particle number, in agreement with equation (10b).

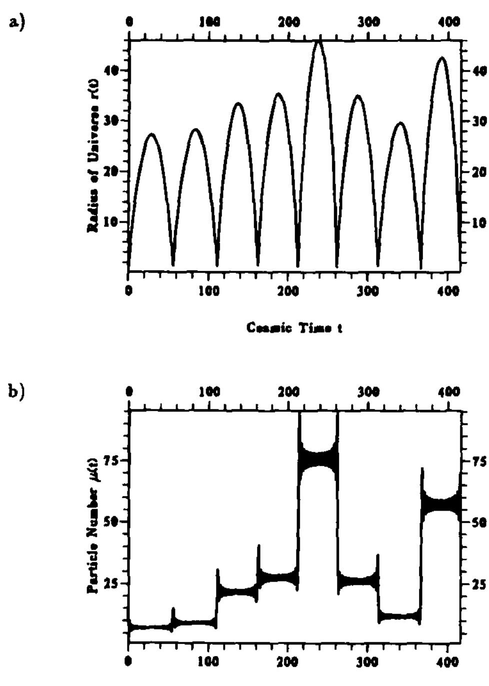


Fig. 3. Maximal radius and particle number. The order relation for the maximal values (per cycle) of (a) the radius r is the same as that for (b) the particle numbers. This indicates that the particle number can *increase* only in a *growing* universe (choice of parameters: $\Lambda = 0.5$; $k_0 = -0.001$; $\mu_C = 1.0$).

Through this result, we have to face the problem of the largest achievable particle number. If the correlation mentioned just above is strictly valid, the particle number could in principle be raised *ad infinitum* by continued bouncing with ever-increasing maximal radius per cycle. Besides this type of solution, represented by Fig. 2, the numerical integrations demonstrate the emergence of a further type (Fig. 4). After the maximal radius per cycle has reached its absolute extremal value $r_{\max}$ in agreement with (10b), it decreases again (Fig. 4a) and thus establishes a cosmic supercycle. Similarly, the particle number is increased through any bounce up to roughly 10^4 particles per 3-cell (Fig. 4b) but is then decreasing again, concomitantly with the maximal radius per single cycle. This result again confirms the close correlation between increase (decrease) in particle number and growing (shrinking) of the universe. In any case, the initial-value equation (9b) says for both types

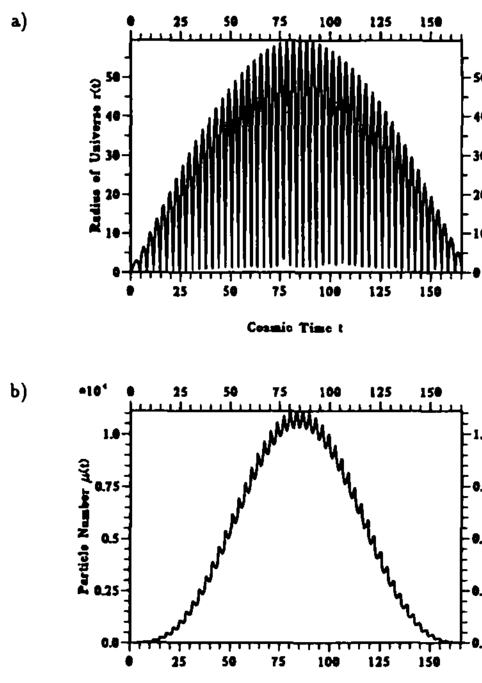


Fig. 4. Cosmic supercycle. (a) The growth of the universe via continuous bouncing reverts to a shrinking phase when the particle production is no longer sufficient to dominate the negative-energy contribution of the vacuum (15). (b) The correlation between growing of the universe and increase of particle number implies the occurrence of a maximal number $\mu_{\max}$ per supercycle, presently $\approx 10^4$ particles per 3-cell in agreement with (10b): $r_{\max} \approx 60$ ($k_0 = -0.05$; $\Lambda = \mu_C = 1$).

of solutions that the universe can become large only if it simultaneously produces sufficient matter.

It is instructive also to reconsider the interplay of matter and the size of the universe from a more physical point of view: In the vicinity of the maximal extension during a given cycle, the particle density ρ is found from (10b) as

$$\rho \approx \frac{\mu_{\max}}{\mathcal{R}_{\max}^3} = m^3 \frac{\mu_{\max}}{r_{\max}^3} \approx m^3 |k_0| = \frac{|\lambda_0|}{Mc^2} \tag{14}$$

and thus is closely related to the cosmological constant λ_0. As a consequence, the energy density $\mathcal{M}$, (7a), of the spinor field ψ is just in the same order of magnitude as the vacuum energy $\mathcal{M}_0$, (6), namely

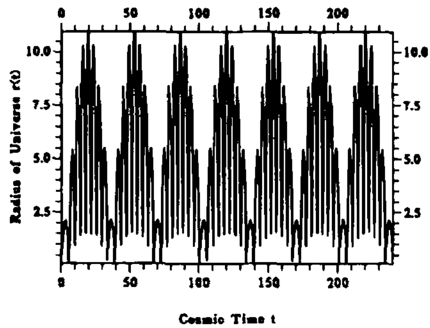
$$\mathcal{M} \approx 3\hbar c\rho \frac{m}{3} = Mc^2\rho \approx |\lambda_0| \equiv |\mathcal{M}_0| \tag{15}$$

This result says that, for our model universe, the vacuum is of the same relevance as the matter content! But Fig. 4 with the result (14) even suggests a more far-reaching conclusion, namely that our model universe can grow only up to that stage where the decreasing energy density of matter becomes of the order of magnitude of the vacuum energy density $|\lambda_0|$, and then the universe has to shrink again. (Observe that for Fig. 4 the initial density $\rho(t \to 0) = m^3\mu_C = \rho_C$ was one particle per Compton 3-volume, $\mu_C = 1$; whereas the density (14) for maximal extension is $\rho \approx m^3|k_0| = 0.05m^3 = 0.05\rho_C$). Obviously our universe tends to avoid the development of a negative total energy density (of matter plus vacuum), i.e., it begins to contract soon enough in order to escape the dominance of the vacuum energy over ordinary matter! The other possibility for circumventing that vacuum dominance would consist in producing more particles; and we want to clarify now why this is somewhat difficult to achieve for our model universe.

5. PERIODIC EVOLUTION

Moreover, we have to ask also what comes at the end of a cosmic supercycle. Is there "extinction into *nihilo*"? This question is difficult to answer by means of numerical integrations. What we did observe is a (more or less) periodic continuation of those supercycles (Fig. 5). Intuitively, one will expect that the "extinction-into-*nihilo*" solutions form a certain subset of the more general supercycle solutions, which predict a kind of (almost?) periodic history for the universe. In what follows, we shall be satisfied with a qualitative understanding of how the periodicity of the supercycles comes about, which represents an alternative to the infinite growth.

a)



b)

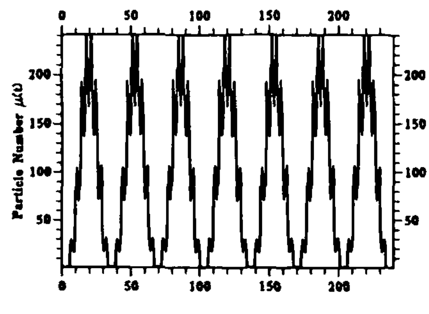


Fig. 5. Periodicity of supercycles. Continuation of the numerical integrations yields a periodic structure for both (a) radius $r(t)$ and (b) particle number $\mu(t)$. The extremal values for μ must occur at phase angles $\chi_l = l\pi$ ($l = 0, 1, 2, 3, \ldots$). This enables the determination of the phase increment $\Delta\chi$ for a supercycle.

It is easy to show (Ochs and Sorg, 1993), and was already mentioned above, that the particle number μ is increased during a bounce when the bounce angle χ_b ($\equiv \chi|_{t_b}$) at the bounce time t_b (i.e., $\dot{r}|_{t_b} = 0$) is in the range $\pi/2 < \chi_b < \pi$ (mod 2π), whereas the particle number is decreased for $\pi < \chi_b < 3\pi/2$ (mod 2π). Thus, it depends upon the increment $\Delta_n\chi = \chi_b^{(n+1)} - \chi_b^{(n)}$ of the phase angle χ between the two bounces at times $t_b^{(n+1)}$ and $t_b^{(n)}$ whether the consecutive bounce at time $t_b^{(n+1)}$ comes just right in the first half (second half) of the total bounce interval $\pi/2 < \chi < 3\pi/2$ (mod 2π) in order that the particle number is increased (decreased). Now, for $r >> 1$, the equation of motion (8a) for χ yields the approximate increment

$$\Delta_n\chi \approx 2(t_b^{(n+1)} - t_b^{(n)}) \tag{16}$$

provided the radius is large enough for most of the time of the cycle. Therefore, for large enough separation of the bounces, i.e.,

$$\Delta_n t_b = t_b^{(n+1)} - t_b^{(n)} >> \pi \tag{17}$$

one can always manage, by suitable choice of the initial conditions, to bring $\chi_b^{(n)}$ for many (all?) bounces into the first half of the bounce interval, eventually even at exactly the same place of this semi-interval ($\Rightarrow \Delta_n\chi = 0 \bmod 2\pi$) in order to raise the particle number for any one of the consecutive bounces. This situation applies for Figs. 2 and 3, where $\Delta_n t_b \approx 60$.

However, if the increment $\Delta_n\chi$ is of the order of the range of the bounce interval π, i.e., when we have $\Delta_n t_b \lesssim \pi$ in place of the condition (17), then it becomes difficult (or even impossible) to have the ideal value $\Delta_n\chi = 0$ (mod 2π) for particle production. This situation is encountered in the case of Fig. 5, where $\Delta_n t_b \approx 4$. As a consequence, the increment $\Delta_n\chi$ becomes

$$\Delta_n\chi = \frac{2\pi}{N} \tag{18}$$

or more generally

$$\Delta_n\chi = \frac{2\pi}{N} \pmod{2\pi} \tag{19}$$

so that one must await an integer number N of bounces in order to have the optimal phase shift $\Delta\chi = N\Delta_n\chi = 0$ (mod 2π). However, this optimal phase shift cannot be fully exploited for matter production, because only the first half $N/2$ of bounces fall into the "matter-producing interval" $\pi/2 < \chi_b < \pi$, whereas the second half $N/2$ fall into the "matter-annihilating interval" $\pi < \chi_b < 3\pi/2$. Thus, the matter produced in the first half of the supercycle is eaten up again during the second half and the total supercycle will consist of N ordinary cycles. Because of the equation of motion for the particle number μ, (8b), the total phase shift $\Delta\chi$ for a supercycle can easily be read off from the μ/t diagram; for instance, for Fig. 5b we have $\Delta\chi = 17\pi - \pi$

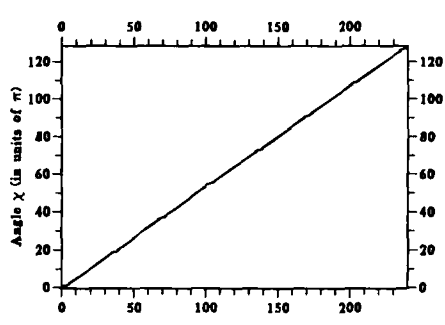


Fig. 6. Linearity between χ and t. The supercycle solutions (Fig. 5) obey an almost linear law between phase angle χ and cosmic time t, which indicates the existence of strictly periodic solutions.

$= 16\pi$. On the other hand, the r/t diagram (Fig. 5a) says that we have eight ordinary cycles during that phase increment, $\Delta\chi = 16\pi$. Consequently, the phase shift per individual cycle is $\Delta_n\chi \cong 16\pi/8 = 0 \pmod{2\pi}$. This yields for the lifetime $\Delta t = \Delta\chi/2$, (16), of an individual cycle $\Delta_n t \approx 16\pi/16 = \pi$, in rough agreement with the r/t diagram of Fig. 5a.

Moreover, the hypothesized linear relationship between cosmic time t and phase angle χ may also be checked by a direct numerical integration (Fig. 6). The fact that the observed linearity is almost perfect suggests that strictly periodic solutions will indeed exist (a rigorous proof of this conjecture is not yet known to us).

6. DISCUSSION

Surely, the present Dirac–Einstein model of the universe is not in agreement with what we observe today around us, because the matter content of the universe is presently not in some globally coherent quantum state ψ. However, the present model could perhaps represent a good simulation of the primeval universe with violent fluctuations of radius $\mathcal{R}$ and energy content $\mathcal{M}$ (we have fixed the topology, which therefore cannot participate in those fluctuations). If one adopts the occurrence of a phase transition (Dolgov *et al.*, 1990), which brings the cosmological constant down to zero at a time when the universe has become large enough and equipped with enough matter, then it will further follow the standard cosmological model. Unfortunately, there is presently no general agreement among cosmologists (Hawking, 1990; Penrose, 1990) about the precise nature of such a phase transition from the quantum era to the standard phase, nor is there a general agreement about the present value of the cosmological constant (Weinberg, 1989). In any case, the standard model needs some modifications for the primeval era and the present Dirac–Einstein theory may be considered as a possible candidate, which additionally is a reasonable alternative to the idea of inflation.

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